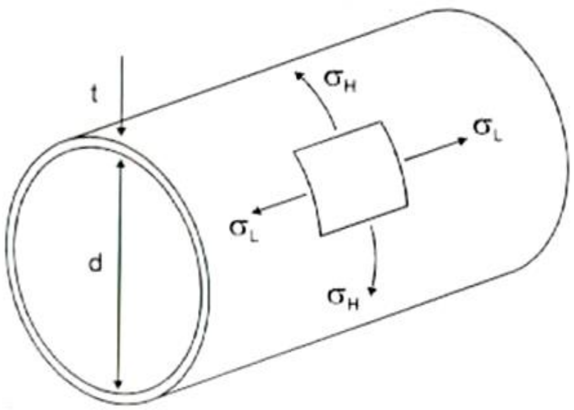
Stress in Cylinder:

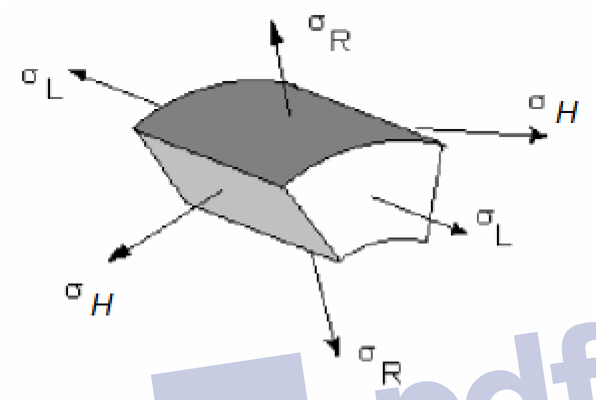
A pressure vessel is a container that holds a fluid (liquid or gas) under pressure, like, propane tanks, and water supply pipes.

Cylinders can be divided to the thin and thick cylinder. Here we will study the stresses in the thin cylinder (such as pressure vessels). In thin wall cylinders the wall thickness is less than 1/10 of the radius of the container.

When a thin–walled cylinder is subjected to internal pressure, three mutually perpendicular principal stresses will be set up in the cylinder.



a) Longitudinal stress σ_L.
b) Circumferential or hoop stress σ_H.
c) Radial stress σ_r.



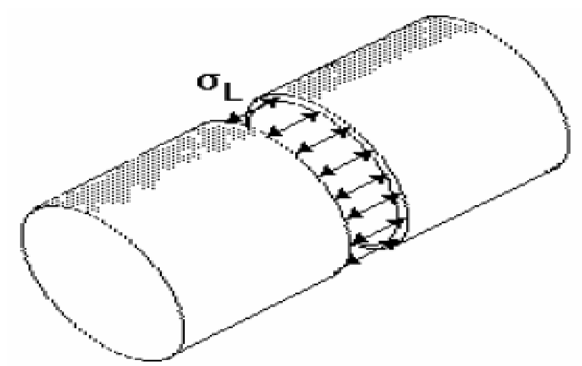
Longitudinal (Axial) stress: is a normal stress parallel to the axis of cylindrical symmetry:

If the pipe has a cap on the end, pressure would push the cap off the end. If the cap is firmly attached to the pipe, then a stress develops along the length of the pipe to resist pressure on the cap.

Imagine cutting the pipe and pressurized fluid transversely. The force exerted by the fluid equals the force along the length of the pipe walls. Pressure acts on a circular area of fluid, so the force exerted by the fluid is:

$$F_{fluid} = P \cdot A$$

$$F_{fluid} = P \cdot \frac{\pi \cdot di^2}{4} \quad \text{or}$$



$$F_{fluid} = P \cdot \frac{\pi\,(2R)^2}{4} \quad \text{or}$$

$$F_{fluid} = P \cdot \pi \cdot R^2$$

We can estimate the cross-sectional area of a thin-walled pipe pretty closely by multiplying the wall thickness by the circumference, as shown in fig. a:

So: $A \approx \pi \cdot di \cdot t$ or

$A \approx \pi \cdot 2R \cdot t$

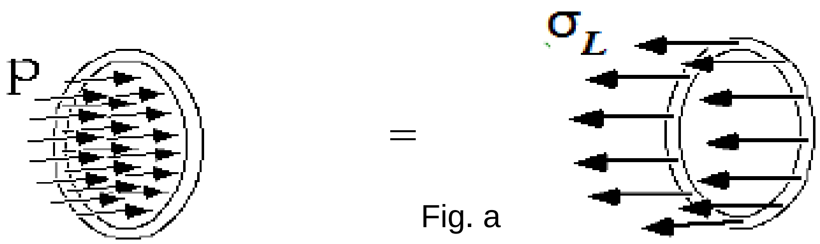


Fig. a

$A = \pi R^2$ $\qquad$ $A = 2\pi R \cdot t$

$F = p(\pi R^2)$ $\qquad$ $F = \sigma_L (2\pi R t)$

$$P(\pi R^2) = \sigma_L (2\pi R t)$$

The stress along the length of the pipe is:

$$\sigma_L = \frac{P \cdot R}{2 \cdot t}$$

Where: σ_L: Longitudinal stress

P: is the internal pressure

R: is the internal radius = d/2 , (where d: is the inside diameter of the cylinder)

t : it the wall thickness

Circumferential stress or hoop stress (σ_H):

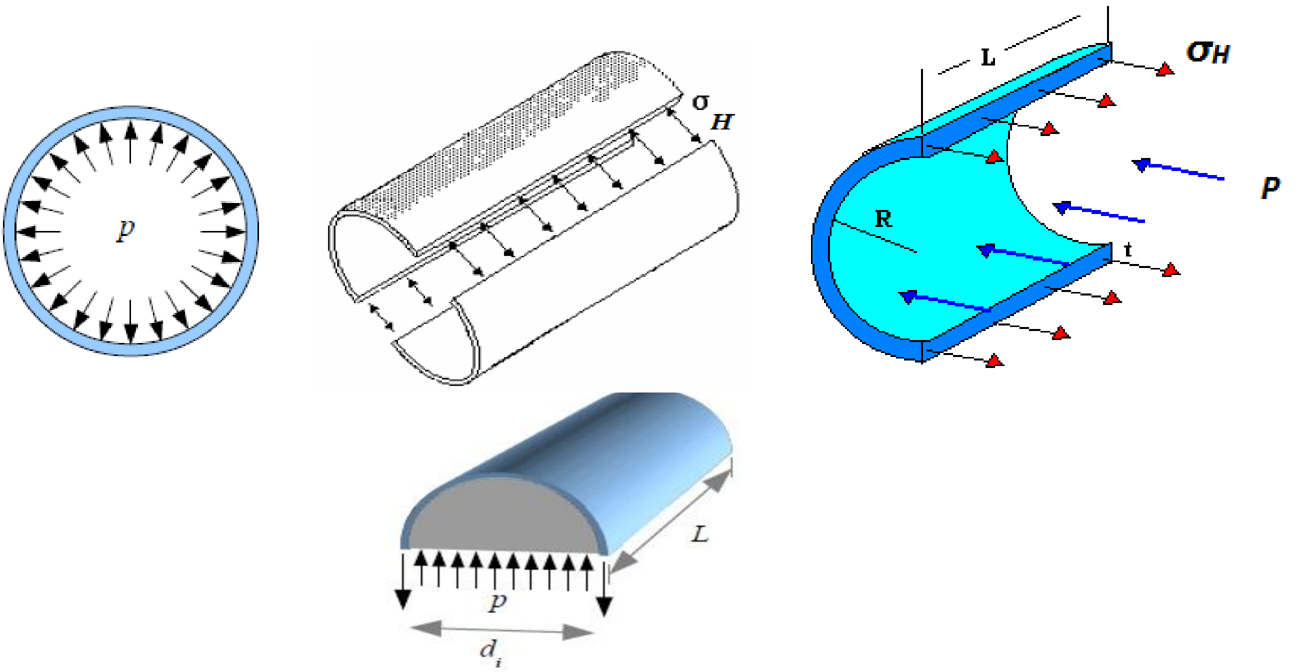


Figure b

Imagine cutting a thin-walled pipe lengthwise through the pressurized fluid and the pipe wall: the force exerted by the fluid must equal the force exerted by the pipe walls (sum of the forces equals zero).

The stress in the walls of the pipe is equal to the fluid force divided by the cross-sectional area of the pipe wall. This cross section of one wall is the thickness of the pipe, *t*, times its length. Since there are two walls, the total cross-sectional area of the wall is $2t\,L$. The stress is around the circumference or the "hoop" direction,

The force exerted by the fluid is $\sigma_H \cdot A = P\,di\,L$

$$\sigma_H \cdot 2t \cdot L = P \cdot 2R \cdot L$$

$$\sigma_H = \frac{P \cdot R}{t}$$

Notice that the length cancels

where: σ_H : circumferential stress or hoop stress

d_i : is the inside diameter of the pipe, and

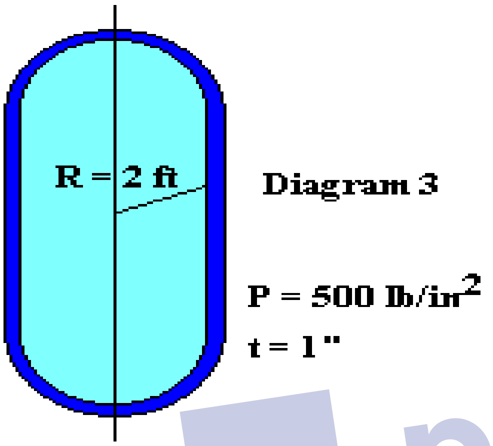
L : is the length of the pipe

R : Radius of the pipe

Radial stress: a stress perpendicular to the symmetry axis.

The radial stress for thin vessels is so small in comparison with the hoop and longitudinal stress that it can be neglected. This is obviously an approximation since, in practice; it will vary from zero at the outside surface to a value equal to the internal pressure at the inside surface.
In Inch-pound-second system (IPS) units for *P* are in pounds-force per square inch (psi). Units for (*t*), and (*d*) are in inches (in). SI units for *P* are in Pascals (Pa), while *t* and *d* =2*r* are in meters (m).

Example: A thin wall pressure vessel is shown in Diagram 3. It's cylindrical section has a radius of 2 feet, and a wall thickness of the 1″. The internal pressure is 500 lb/in^2. Determine the longitudinal and hoop stresses in the cylindrical region.



Solution:

We apply the relationships developed for stress in cylindrical:

$$\sigma_L = P\ R\ /\ 2\ t = 500\ lb/in^2 * 24''/2 * 1'' = 6000\ lb/in^2.$$

$$\sigma_H = P\ R\ /\ t = 500\ lb/in^2 * 24''/\ 1'' = 12{,}000\ lb/in^2.$$

Example:

Consider a cylindrical pressure vessel with a wall thickness of 25 mm, an internal pressure of 1.4 MPa, and an outer diameter of 1.2 m. Find the axial and tangential stresses.

Solution:

$$q = 1.4\ \text{MPa};\ D = 1200 - 50 = 1150\ \text{mm};\ t = 25\ \text{mm}$$

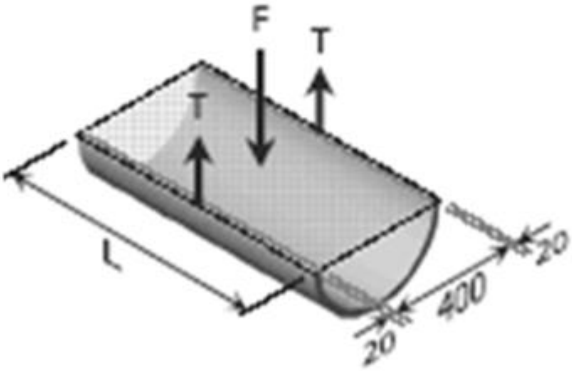
$$\sigma_t = \frac{qD}{2t} = \frac{1.4\ \text{MPa} \times 1150\ \text{mm}}{2 \times 25\ \text{mm}} = 32.2\ \text{MPa}$$

$$\sigma_a = \frac{qD}{4t} = \frac{1.4\ \text{MPa} \times 1150\ \text{mm}}{4 \times 25\ \text{mm}} = 16.1\ \text{MPa}$$

Example:

A cylindrical steel pressure vessel 400 mm in diameter with a wall thickness of 20 mm, is subjected to an internal pressure of 4.5 MN/m2. (a) Calculate the tangential and longitudinal stresses in the steel. (b) To what value may the internal pressure be increased if the stress in the steel is limited to 120 MN/m2?

Solution:



(a) Tangential stress (**Hoop Stress**) :

$$F = 2T$$

$$pDL = 2(\sigma_H tL)$$

$$\sigma_H = \frac{pD}{2t} = \frac{4.5(400)}{2(20)}$$

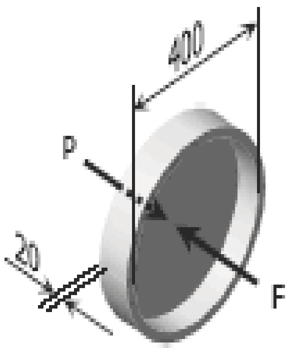
$$\sigma_H = 45 \text{ MPa}$$

Longitudinal Stress :

$$\tfrac{1}{4}\pi D^2 p = \sigma_L(\pi D t)$$

$$\sigma_L = \frac{pD}{4t} = \frac{4.5(400)}{4(20)}$$

$$\sigma_L = 22.5 \text{ MPa}$$



(b) From (a), $\sigma_H = \dfrac{pD}{2t}$ and $\sigma_L = \dfrac{pD}{4t}$ thus, $\sigma_H = 2\sigma_L$

this shows that tangential stress (**Hoop Stress**) is the critical.

$$\sigma_H = \frac{pD}{2t}$$

$$120 = \frac{p(400)}{2(20)}$$

$$P = 12 \text{ MPa}$$